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System Optimization of Ablative Pulsed Plasma Thrusters for Stationkeeping

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We present a realistic and easy-to-apply model for the system optimization of ablative pulsed plasma propulsion for stationkeeping missions. The model yields the characteristics of the propulsion module(s) that lead to minimum propulsion system mass for given mission specifications. We show that using an empirical relation for mass production, an expression relating the optimal (minimum) propulsion system mass to the thruster's most important scaling parameter E/I (the ratio of the discharge energy E to the impulse bit I) can be found and, remarkably, is independent of the state of capacitor and power conditioning technologies. This independence allows unfolding the relations between the module mass, E/I, and the mission requirements in a single plot that is applicable for a wide range of delta-v and payload masses. The use of the model to characterize the optimal design of a pulsed plasma thruster system is illustrated with the example of a 10-year north–south stationkeeping of a medium size commercial satellite at geosynchronous orbit. We show that even with off-the-shelf capacitors, the use of such thrusters can result in propulsion mass fractions as low as 6.4%.

Nomenclature

A	= exposed propellant area, m^2
c	= feed configuration constant, nondimensional
d	= feed configuration constant, nondimensional
E	= discharge energy, J
E_i	= ratio of discharge energy to impulse bit (power-to-thrust ratio), J/N · s or W/N
$F(E_i)$	= propellant production function, kg/J
f	= pulse frequency, Hz
g	= gravitational acceleration, m/s^2
I	= impulse, bit, N · s
I_s	= effective specific impulse, s
I_t	= total impulse, N · s
M_{cb}	= mass of capacitor bank, kg
M_{fix}	= fixed mass, kg
M_p	= mass of one propulsion module, kg
M_{pack}	= packaging mass, kg
M_{pc}	= mass of power conditioning system, kg
$M_{\text{pc}_{\text{fix}}}$	= power-independent mass of power conditioner, kg
M_{satp}	= mass of satellite payload, kg
M_{sp}	= mass of solid propellant, kg
M_{tot}	= total mass of satellite, kg
$\tilde{m}_{\text{cb}}$	= capacitor (or capacitor bank) specific mass, kg/J
$\tilde{m}_{\text{pc}}$	= power conditioner specific mass, kg/W
N_m	= number of propulsion modules
N_p	= number of pulses
P_b	= required bus power, W
Δv	= mission delta-v, m/s
ϵ	= packaging ratio, nondimensional
η	= propulsion system total efficiency, nondimensional
η_{pc}	= efficiency of power conditioner, nondimensional
κ	= ablation constant, kg/J
Ψ	= mass scaling function, nondimensional

I. Introduction

THE trend toward small satellites in the military, commercial, and scientific sectors and the prospect of the near-term deployment of small satellite constellations, call for a re-evaluation of propulsion options that are particularly suited and beneficial to power and mass-limited satellites. Stationkeeping propulsion subsystems on small satellites are usually the most massive subsystems and, consequently, mass-saving high-specific impulse electric propulsion is emerging as a competitive alternative. The mass savings afforded by electric thrusters translate directly into cost savings mainly through the use of smaller launchers or the possibility of mounting multiple satellites on a single launcher.

Often, small satellites are power limited to levels below 300 W. At these power levels and below, only the ablative pulsed plasma thruster (APPT) can operate nominally. Indeed, APPT systems operating at power levels of a few tens of watts have been designed and flown successfully.[1] (See Ref. 1 for a description of the APPT and its characteristics.)

In this paper we look at the problem of optimizing the design of an APPT system for given satellite mass and mission requirements. By optimizing we mean choosing the APPT system parameters that result with the smallest mass fraction (M_p/M_{tot}), given the mission requirements and the state of related technologies. Using present technology parameters, we illustrate the optimization model by quantifying this mass fraction for a typical stationkeeping mission.

We do so by starting with a general but realistic model for the optimal APPT system mass that includes four general types of masses: energy-dependent mass, power-dependent mass, fixed mass, and size-dependent mass. The optimization model is cast in terms of the most important scaling parameter E/I (where E is the discharge energy and I the impulse bit). We then specialize the model with an empirical relation for the mass production rate and find that the mass of an optimized system as a function of E/I is independent of the state of capacitor and power conditioning technologies. The resulting prescription allows for a straightforward application of the model to a spectrum of mission requirements of current interest. Finally, we illustrate the use of the model for the design of an APPT system optimized for the 10-year north–south stationkeeping of a medium-size commercial satellite at geosynchronous orbit (GEO).

II. General Model

Express the total mass M_{tot} of the satellite as

$$M_{\text{tot}} = N_m M_p + M_{\text{satp}} \tag{1}$$

The mass of one APPT propulsion module, M_p, can be broken down into the following components:

$$M_p = M_{\text{cb}} + M_{\text{pc}} + M_{\text{sp}} + M_{\text{fix}} + M_{\text{pack}} \tag{2}$$

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where the fixed mass $M_{\rm fix}$ includes the mass of the electrodes and ignitor assembly. The mass of the packaging $M_{\rm pack}$ can be expressed as a fraction ϵ of the total module mass

$$M_{\rm pack} = \epsilon M_p \tag{3}$$

The mass of the capacitor bank mass can in turn be expressed in terms of the discharge energy E and the specific mass $\bar{m}_{\rm cb}$ of the capacitors (in kilograms per Joule),

$$M_{\rm cb} = \bar{m}_{\rm cb} E \tag{4}$$

The mass of the power conditioning system can be written as the sum of a fixed mass and a power-dependent mass,

$$M_{\rm pc} = M_{\rm pc_{fix}} + \bar{m}_{\rm pc} P_b \tag{5}$$

where $M_{\rm pc_{fix}}$ is the power-independent mass of that system that we include, along with other fixed masses of the system, in $M_{\rm fix}$. Also, $\bar{m}_{\rm pc}$ is the specific mass of the power conditioning system (in kilograms per watt) and P_b is the power required from the spacecraft bus. This power is related to the discharge energy, the pulse frequency f, and the power conditioning efficiency $\eta_{\rm pc}$ by

$$P_b = f E / \eta_{\rm pc} \tag{6}$$

Finally, the mass of the solid propellant can be expressed as

$$M_{sp} = N_p F(E_i) E \tag{7}$$

where N_p is the number of pulses and $F(E_i)$ is propellant production function (equivalent to the specific mass of ablated solid propellant per shot) that has been shown by experiments to be a function of the ratio E_i of the discharge energy E to the impulse bit I,

$$E_i \equiv E/I \tag{8}$$

A stationkeeping mission can be characterized in terms of a Δv that for a certain satellite mass would require a total impulse I_t from the pulsed propulsion system

$$I_t = N_m N_p I = M_{\rm tot} \Delta v \tag{9}$$

where I is the impulse bit. This equation carries the implicit assumption that the total mass does not change appreciably over the time of the entire mission, which is equivalent to saying

$$M_{\rm sp} \ll M_{\rm tot} \tag{10}$$

This assumption must, therefore, be checked a posteriori to validate the calculations.

Combining Eqs. (1–9) we can write an expression for the mass of one APPT module

$$M_p = \frac{\Psi (M_{\rm satp}/N_m) + M_{\rm fix}}{1 - \epsilon - \Psi} \tag{11}$$

where Ψ is a mass scaling function given by

$$\Psi = \frac{\Delta v}{N_p} \left[\bar{m}_{\rm cb} + \frac{f \bar{m}_{\rm pc}}{\eta_{\rm pc}} + N_p F(E_i) \right] E_i \tag{12}$$

With the preceding two equations, the APPT module mass is expressed as a function of the energy-to-impulse bit ratio E_i, which is an important parameter in scaling the APPT performance.

For a given state of the capacitor and power conditioning technology, the parameters $\bar{m}_{\rm cb}, \bar{m}_{\rm pc}$, $\eta_{\rm pc}$, and the highest number of pulses N_p are fixed. Under such conditions there is for each value of f an optimal value E_i for which M_p is minimum. We find a relation between f and E_i at this optimal condition by setting $\partial M_p / \partial E_i$ to zero and solving for f^* to obtain

$$f^* = -\frac{\eta_{\rm pc}}{\bar{m}_{\rm pc}} \left\{ \bar{m}_{\rm cb} + N_P \left[F(E_i) + E_i \frac{\partial F(E_i)}{\partial E_i} \right] \right\} \tag{13}$$

The optimal discharge energy is then given by

$$E^* = \frac{M_p(1-\varepsilon) - M_{\rm fix}}{\bar{m}_{\rm cb} + \left(f^* \bar{m}_{\rm pc} / \eta_{pc} \right) + N_p F(E_i)} \tag{14}$$

Because we have expressed the mass model in terms of E_i it is also convenient and straightforward to express the following quantities in terms of E_i.

The optimal specific impulse I_s^*:

$$I_s^* = \left[E_i^* F(E_i) g \right]^{-1} \tag{15}$$

The total system efficiency at optimal conditions:

$$\eta^* = \eta_{\rm pc} \left[2 E_i^* F^2(E_i) g \right]^{-1} \tag{16}$$

III. Model Specialized with a Mass Production Law

We now specialize the general model for a specific function $F(E_i)$ obtained experimentally. It was found in Ref. 2 that the following empirical relation holds for the mass production of the propellant plasma:

$$F(E_i) = E_i^{-4} / 4\kappa \tag{17}$$

where κ is an ablation constant that depends only on the propellant. For Teflon®, $\kappa = 4 \times 10^{-11}$ kg/J. Equation (17), being empirical, is strictly valid for the range of experimental parameters investigated in Ref. 2 (1×10^{-3} N-s $< I < 5 \times 10^{-3}$ N-s). The extent to which this relation can be generalized or altered to cover parts of the parameter space outside that studied in Ref. 2 must be established through future research on APPT scaling.

Using Eq. (17) for $F(E_i)$ in Eq. (13), we obtain the following expression for f^*:

$$f^* = (\eta_{\rm pc}/\bar{m}_{\rm pc}) \left[(3N_p/4\kappa) E_i^{-4} - \bar{m}_{\rm cb} \right] \tag{18}$$

The optimal value for the mass scaling parameter becomes

$$\Psi^* = (\Delta v/\kappa) E_i^{-3} \tag{19}$$

Note from Eq. (19) that Ψ^* becomes independent of all of the specifications of the propulsion subsystem (specific masses, power conditioning efficiency, and number of pulses). This means that the optimal propulsion module mass, given by

$$M_p^* = \frac{\Psi^* (M_{\rm satp}/N_m) + M_{\rm fix}}{1 - \epsilon - \Psi^*} \tag{20}$$

can be expressed as a function of E_i that depends only on the following four parameters: Δv, $M_{\rm satp}/N_m$, $M_{\rm fix}$, and ϵ. The first two parameters are dependent on the mission and satellite requirements whereas the last two, the fixed mass and the packaging coefficient, are independent of the capacitor and power conditioning technologies.

$M_{\rm fix}$ can be estimated from experience[3] as the sum of the mass of the discharge ignitor and its circuitry (0.23 kg), the mass of the electrodes and the associated assembly (3 kg), and the mass of the power independent part of the power conditioning system (0.5 kg) giving $M_{\rm fix} = 3.75$ kg. Whereas the packaging ratio ϵ for more complex electric propulsion systems, such as ion and magnetoplasma dynamic (MPD) thrusters, can be as high as 0.5 (see Refs. 4 and 5) APPT flight-ready prototypes typically have a packaging ratio of $\epsilon = 0.2$ (see Ref. 2).

With fixed values of $M_{\rm fix}$ and ϵ, Eqs. (19) and (20) can be used with Ψ^* as an intermediate parameter to build a graph that allows finding M_p^* as a function of E_i with the mission requirements ($M_{\rm satp}/N_m$ and Δv) as parameters. This plot is shown in Fig. 1 for an interesting range of these parameters. The arrows across the plot show how the curves are used for the case study discussed later. This plot is independent of capacitor and power conditioning technologies. The use of this graph to design an optimal APPT system is illustrated in the case study.

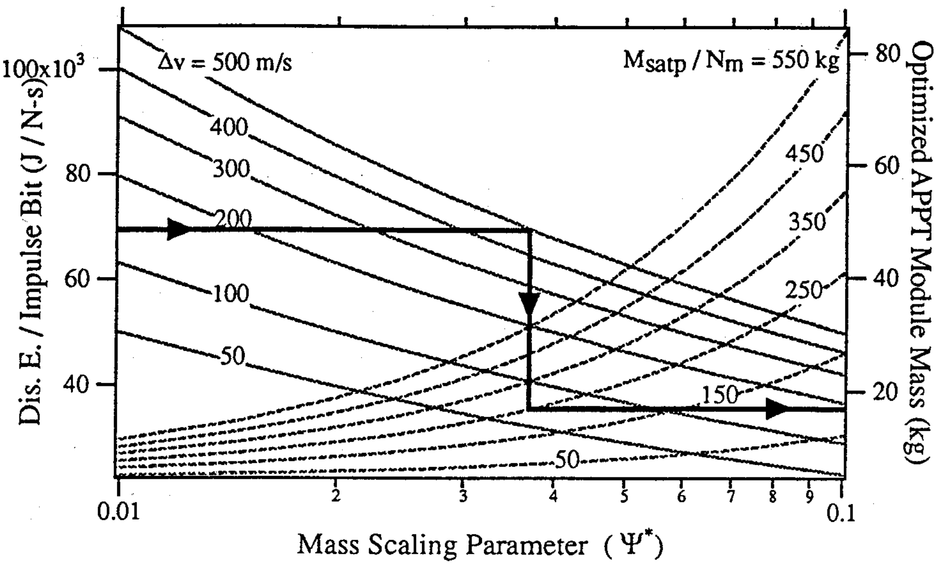

Fig. 1 **Relation of the optimized APPT module mass M_p^* to the ratio of discharge energy to impulse bit E_i.**

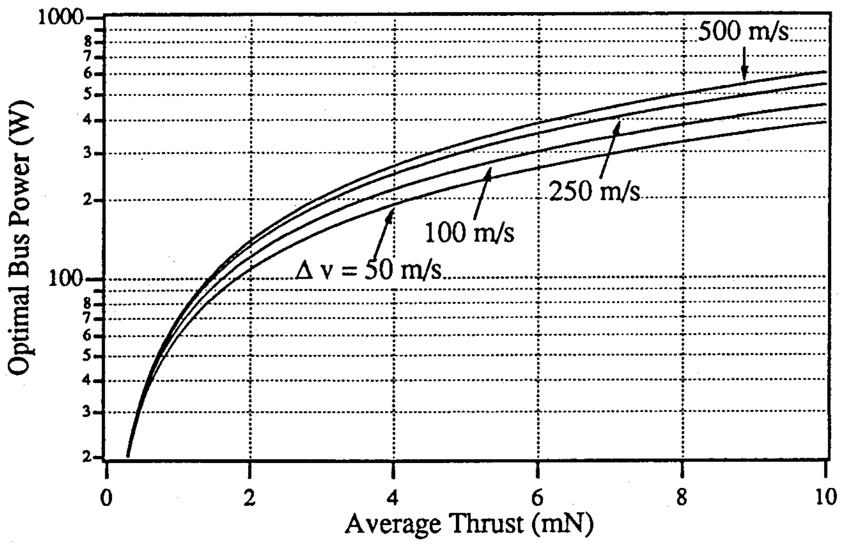

Fig. 2 **Optimal bus power vs desired average thrust level.**

IV. Model Further Specialized with State of the Capacitor Technology

For a given state of the capacitor (and power conditioning) technology, a series of curves can be drawn using the described model that gives the relation between the desired average thrust and the required power from the spacecraft bus for a range of mission requirements. For our model calculations, we assume the state of capacitor (and power conditioning) technology to be represented in the following sections.

A. Capacitor Specific Mass

Capacitor technology has made great advances since the early development time of the APPTs in the 1970s. The capacitors with the highest energy density $\bar{m}_{\text{cb}}^{-1}$ that are presently available are aluminum electrolytic capacitors[6] with $\bar{m}_{\text{cb}}^{-1}$ as high as 99 J/kg but are currently deemed unreliable for long-term space applications.[4] The double-layer capacitor technology promises a two order of magnitude improvement in the energy density, but these capacitors are still in the research stage.[7,8] Barium-strontium titanate and lead-zirconate titanate capacitors are being developed with energy densities of 10^5 J/kg but have not yet been shown to be suited for long-term APPT applications. Ceramic capacitors are an off-the-shelf technology and offer a factor of two improvement on the energy densities of the capacitors used on early APPT systems. They have been recently chosen in a pulsed MPD propulsion system study[4] and are benchmarked at 50.3 J/kg for the ceramic Z5U capacitor.[9] For our current study, we set $\bar{m}_{\text{cb}}^{-1} = 50$ J/kg.

B. Power Conditioner Specific Mass and Efficiency

The power conditioner (PC) for APPT systems is much simpler than that required for other EP systems and is not as critical as the capacitor bank. The power conditioner technology of the mid-1970s[3] is still suited for today's APPT applications and is characterized by a fixed mass of 0.5 kg (added to M_{fix} in the given model), a specific mass $\bar{m}_{\text{pc}} = 8 \times 10^{-3}$ kg/W and an efficiency of $\eta_{\text{pc}} = 8$.

C. Number of Pulses

The largest number of pulses demonstrated for an APPT system is in the 10^7 range and is limited mostly by capacitor failure. High-energy density capacitors have not yet been demonstrated to exceed that range and so we take $N_p = 3 \times 10^7$ for our present study. The chosen technology parameters are summarized in Table 2. The plot in Fig. 2 gives the required bus power (which must not exceed the available bus power) as calculated by the model for the optimal characterization of one APPT module at a desired average thrust level for the case of $M_{\text{satp}}/N_m = 250$ kg and a range of Δv of interest. The following parameters were fixed: $\bar{m}_{\text{cb}}^{-1} = 50$ J/kg, $\bar{m}_{\text{pc}} = 8 \times 10^{-3}$ kg/W, $\eta_{\text{pc}} = 0.8$, and $N_p = 3 \times 10^7$.

For a chosen average thrust level per module and number of modules, the plot gives the required bus power for a given Δv. The corresponding value for the power level will also specify the optimal value of $E_i (E_i = \eta_{pc} P_b / T)$ that can be used in the plot of Fig. 1 to obtain the mass of the module. Finally, the optimal pulse frequency is given by the curves in Fig. 3 that were obtained using Eq. (18). In that plot we set $\bar{m}_{cb}^{-1} = 50$ J/kg, $\bar{m}_{pc} = 8 \times 10^{-3}$ kg/W, and $\eta_{pc} = 0.8$. We note that this plot is independent of the mission requirements (i.e., Δv, M_{satp}, and N_m).

V. Sample Case

A typical example from the spacecraft engineer's point of view is the case of the north-south stationkeeping of a 500-kg payload in GEO for 10 years. We assume that the maximum available bus power is 300 W. The particulars of the mission (stabilization scheme, pointing requirements, stationkeeping allotments, power budgeting, etc.) dictate the choice of the number of modules and the average thrust. We assume that two modules are needed and that each must deliver an average thrust of 1.5 mN. These requirements are summarized in Table 1.

The worst-case changes in velocity required for stationkeeping at GEO are $\Delta v_{moon} = 36.93$ m/s per year and $\Delta v_{sun} = 14.45$ m/s per year that for 10 years yield a total of $\Delta v = 513.8$ m/s. From Fig. 2, for an average thrust of 1.5 mN, we find a required bus power of 105 W that is acceptable because the total power for the two modules (210 W) does not exceed the available power of 300 W. This yields an energy to impulse bit ratio of $E_i = 70 \times 10^3$ J/N-s. At this value of E_i (and for $N_p = 3 \times 10^7$), we find from Fig. 3 that to ensure the optimal conditions the pulse frequency for each module must be 0.3 Hz. Furthermore, for that value of E_i and the required Δv, we find from the left side of the plot in Fig. 1 a mass scaling parameter $\Psi^* = 0.04$ that from the right side of the same plot (with $M_{satp}/N_m = 250$ kg) yields an APPT module mass of about 17 kg. The calculation is illustrated by the arrows drawn across that plot. The total propulsion system mass is, thus, 34 kg. This is 6.4% of the total spacecraft mass. Other pertinent system parameters can be calculated with these relations and are shown in Table 3. In particular, we find the total mass of required Teflon to be 5 kg, which satisfies the assumption that $M_{sp} \ll M_{tot}$.

Table 1 Requirements

Satellite payload mass, kg	500
Mission type	10-year NSSK at GEO ($\Delta v = 513.8$ m/s)
Number of propulsion modules	2
Available bus power, W	300
Average thrust per module, mN	1.5

Table 2 Assumed technology parameters

Capacitor energy density, J/kg	50
Power cond. specific mass	0.5 kg + 0.08 kg/W
Power cond. efficiency, %	80
Maximum number of pulses	3×10^7
Fixed mass, kg	3.73
Packaging mass ratio	0.2

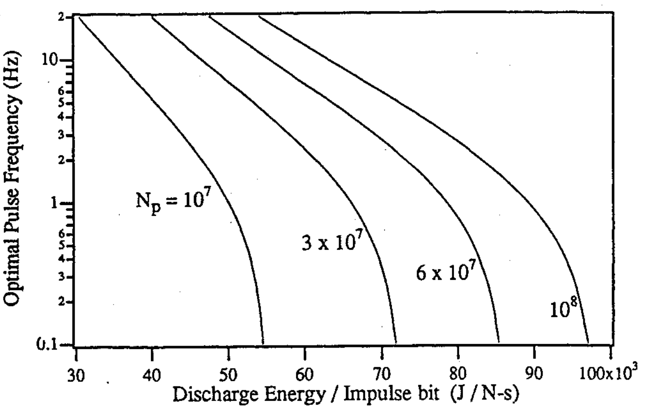

Fig. 3 Optimal pulse frequency as a function of E_i.

Table 3 Optimized APPT characteristics

Required bus power, W	105
Impulse bit, m N-s	4.57
Pulse frequency, Hz	0.33
Discharge energy, J	321
Effective specific impulse, s	5621
Total efficiency	0.31
Geometry	Breech fed
Teflon exposed area, cm^2	3
Mass of capacitors, kg	6.4
Mass of power cond., kg	3
Mass of Teflon, kg	2.5
Total mass of APPT module, kg	17.1

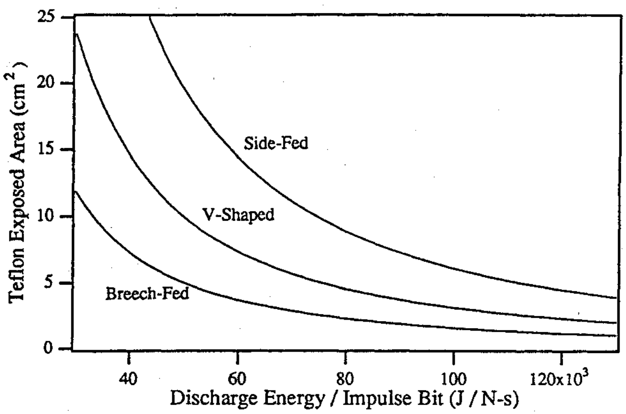

Fig. 4 Teflon exposed area as a function of E_i.

VI. Thruster Design Considerations

By combining fits to experimental data obtained with various thruster designs,[3,10] we can write the following approximate empirical expression that relates E_i to the exposed area A of Teflon in the thruster:

$$A = E_i^{-\frac{1}{n}} (d/c)^{\frac{1}{n}} \tag{21}$$

where all units are in SI and $n = 0.585$, $d = 1.4511$ with c dependent on the particular feed configuration. In particular, $c = 2.49 \times 10^{-3}$ for a breech-fed geometry, $c = 1.659 \times 10^{-3}$ for a V-shaped geometry, and $c = 1.116 \times 10^{-3}$ for a side-fed geometry. This relation is plotted in Fig. 4 for the three feed geometries studied in Refs. 3 and 10.

The breech-fed geometry is preferable when the optimization yields a requirement of high-specific impulse, which is the case for the sample calculation here. For that geometry, and our calculated value of $E_i = 70 \times 10^3$ J/N-s, the optimal exposed area is about 3 cm^2. Considering the uncertainty in the experimental database, this value for the exposed area should be taken only as the starting point for iterating the design of a thruster that satisfies the optimal criteria calculated earlier. The results of this sample optimization calculation are displayed in Table 3.

VII. Concluding Remarks

The unique ability of APPT systems to provide high-specific impulse at low-power levels renders them particularly suited for stationkeeping propulsion on the many small power-limited satellites being foreseen for the near future. Although many APPT systems have reached flight readiness in the late 1960s and mid 1970s, most of these systems, were limited to operation at a few tens of joules of discharge energy. For many realistic stationkeeping requirements, we have shown that optimal APPT operating conditions are reached for discharge energies of a few hundred joules.

The scaling toward higher energy systems and the integration of modern power processing and storage technologies, among other technical improvements, can lead toward systems that can be optimized, as shown in this paper, to yield total mass fractions of a few percent for even the more requiring stationkeeping missions.

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